Programming Design for Multi-layer Mapping

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ABSTRACT

One of the basic parts which constitute the study of spatial information is the map projection. All projections have mathematical formulas which define the relationship between the latitude/longitude graticule on the earth and its representation on the map sheet, or the relationship between the geographic coordinates (latitude and longitude) of points and their projected coordinates (grid or rectangular coordinates) on the map. Adopting this concept, the mathematical derivations of the map projection are used in this paper as the basis for the design of software application which will be called MLM (Multi-Layer Mapping).

The map projection model used here is a product of two conformal projection, these are :

1- Projection of ellipsoid onto a sphere.
2- Projection of the sphere onto a plane.

The actual geographical coordinates of the earth ellipsoid are used in these projections within the MLM in order to display the interesting layers which constitutes the geographical map of the study area on the computer screen, the MLM dose also an inverse calculation of projection, i.e. when a user selects a point on the screen, it displays the corresponding values of the geographical coordinates for the selected point.

The MLM provides the following main functions: display the grid of angles (latitude and longitude) for the study area, display multilayer features choosing by the user, producing geographical spatial information for any point on the map, computing of the distance between any two selected points on the stereographic plane, finally the printing of the study map

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Keyword: stereographic projections, conformal projection, geographical coordinate, GIS, spatial data, ellipsoid.

1. Introduction

The need to place information in a geographical context pervades many aspects of activity. In geodetic concept, many of these activities are concerned with the recording, georeferencing, and analyzing the acquired dataset from the navigation and leveling techniques

[9],[1]. Geodetic mapping and other control surveys can be carried out effectively using high-grade Global Positioning System: GPS equipment and Geographical Information System: GIS software. This technique can set new standards of accuracy and productivity [4].

The purpose of this paper is to focus on two essential things which are of help to the designers of GIS applications; the first one is to select the appropriate formulas with a geographical background (usually a georeferencing data with a selective projection system) and hence deriving new formulae which will be the basis for drawing a multilayer map such as boarders,rivers,cities,....etc., the second one is the spatial referencing process which is very important to produce a geographical coordinate (latitude and longitude) for any point on the study map.

An application program is designed using a complete set of derived formulas with spatial reference, so that a grid of latitude and longitude lines is produced on the computer screen, selected layers of a map can also be drawn on the screen using the same coordinate system adopted on the main program.

The designed program is focused on the Middle East area as a main goal, with a special emphasis on Iraq as a geographical study area in this research to produce the desired layers according to the available dataset which is derived from a geographically database references.

The designed program have the ability to produce an overlay data sets, that reduces the hard work of doing this only with transparencies. The techniques applied in this study are actually depends on the database, map information, and a computer based linked between them. All the subroutines which constitutes the MLM are written in Visual Basic6, this language is quite convenient for this research, it offers all the facilities we need for the design of MLM , beside its widespread and simplicity of use.

The program will be of great help to the workers in the fields of geographical applications, remote sensing, and computer science.

2. Coordinate System and Map Projection

Coordinates are a convenient method of recording position in space. They may be used to locate position in two dimensions, such as a point on a graph. An extension of this method to map usage allows the location of a place to be represented by its grid reference (such as geographical coordinate).

In addition to providing a means of reference, coordinates can also be used as a convenient way for solving certain geometrical problems. The branch of mathematics known as coordinate geometry analyze problems through the relationship between points as defined by their coordinates. By these means, it is possible to derive algebric expressions defining different kinds of curves which can not be done by Euclidean geometry [2].

Since the earth is a rotating body, the obvious datum from which we may define its geometry is its axis of rotation. This surface intersects at two points which are poles to primary great circle whose plane is perpendicular to the axis. This primary great circle is the Equator and its poles are the North and South geographical poles. The secondaries to the equator are not given a single name but the word Meridian describes each semicircle of a pair which together forms a single secondary [7]. However, the plane of the equator is used as the datum for measurement of the vertical angle which we know as Latitude; the other plane is that of the meridian chosen as zero Longitude as shown in figure (1).

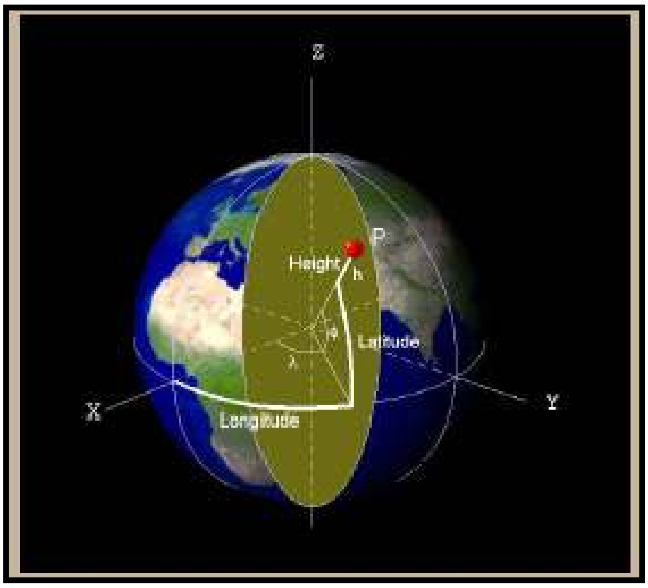

Figure (1): latitude and longitude on the sphere

The latitude of a point defined as the angle measured at the center of the earth between the plane of the equator and the radius drawn to the point. The longitude of a point on the earth's surface represents the second vertorial angle required to define the position, it is defined as the angle measured in the plane of the equator between the plane of the meridian through the point and the plane of some other meridian selected as datum [9].

Coordinate geometry is an exceptionally powerful tool in the study of the theory of map projections and without its help it is practically impossible to pass beyond the elementary descriptive stage.

Map projection is a mathematically described technique of how to represent the curved planets' surface on a flat map. To represent parts of the surface of the earth on a flat paper map or on a computer screen, the curved horizontal reference surface must be mapped into the 2D mapping, i.e.; assigning plane Cartesian coordinates (x, y) to each point on the reference surface with geographical coordinate (latitude, longitude).

Some map projections can be visualized as true geometric projections directly onto the mapping plane, in which case it calls as an azimuthally projection (some time called as planar projection), or onto an intermediate surface, which is then rolled out into the mapping plane. Typical choices for such intermediate surfaces are cones and cylinders. Such map projections are often called conical, and cylindrical, respectively [8]. Figure (2) shows the surfaces involved in these three classes of projections

2-1 Stereographic projection

Cartographers are evaluated the given projection by how it distorts the earth's

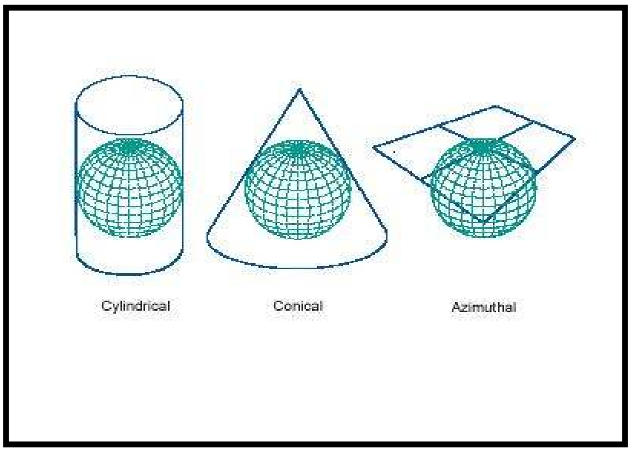

Figure (2): The main three types of the projection

surface during the transformation from a sphere or ellipsoid to a plane surface or flat map. Some projections preserve the property of local shape, so that the outline of a small area like a state or a part of a coastline is correct. These are called "conformal" projection [3]. It is a map projection in which the angles at each point are preserved. This means that the shapes of small areas are maintained accurately. One particular kind of this projection is the stereographic projection.

Stereographic projection is one type of the planar projection, it views the surface data from a specific point in space. The point of view determines how the spherical data is projected onto the flat map or surface [10]. Thus stereographic projection involves the rendering of solid, or three-dimensional objects onto a two dimensional page. It views the surface data from pole to pole by considering that the plane is tangent to the sphere at either North or the South Pole (which will be the center of the system) as illustrated in figure (3) below.

More details of the mathematical basis of this projection will be explained in the next section.

Stereographic projection has two important characteristics that differentiate it from other kinds of projections:

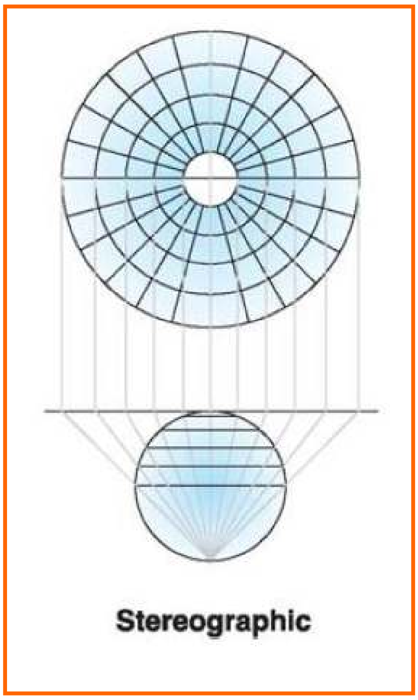

Figure (3): stereographic projection

1- Stereographic Projection preserves circles.

2- Stereographic Projection preserves angles.

This means that circles on a sphere (i.e. latitudes on the Earth) are represented as circles on a plane and the angles between lines are retained when the lines are projected.

In present paper, we concentrate on the stereographic projection that preserves angles in the programming process of the design software.

3. Theoretical Considerations

Map projection is the basis of any software design that is capable of producing a spatial information. In this paper, two projection systems have been applied to fulfill the requirements of the software design:

3-1 Projection of ellipsoid onto a sphere:

The earth is not a sphere but an ellipsoid of revolution, this projection usually replaces the earth ellipsoid by a sphere that approximates the natural surface of the earth in overall shape, especially the general curvature, it preservers the angles (latitude and longitude), meridians, north and the north angle of convergence, this is generally known as conformal projection.

In the terms of the map projection theory, the equations for conformal projection from an ellipsoidal surface into a spherical surface is given by [7]:

$$\tan(\frac{\pi}{4}+\frac{L_c}{2}) = \tan(\frac{\pi}{4}+\frac{L}{2})*(\frac{1-e*\sin(L)}{1+e*\sin(L)})^{e/2} \quad \ldots 1a$$

$$\ell_c = \ell \quad \ldots 1b$$

where:

L = the geodetic latitude, which is the complement of the angle between a right angle to the ellipsoid and the polar axis

ℓ = longitude of a point on the earth ellipsoid, whish is the angle of the meridian plane passing through this point with the original meridian plane (passing through Greenwich).

L_c=conformal latitude

ℓ_c = conformal longitude

This means that at any point on the earth ellipsoid (L,ℓ), there is a corresponding point on the sphere (L_c,ℓ_c). Generally, the sphere is known as the "conformal sphere" and the latitude (L_c) is known as the "conformal latitude" as illustrated in figure (4).

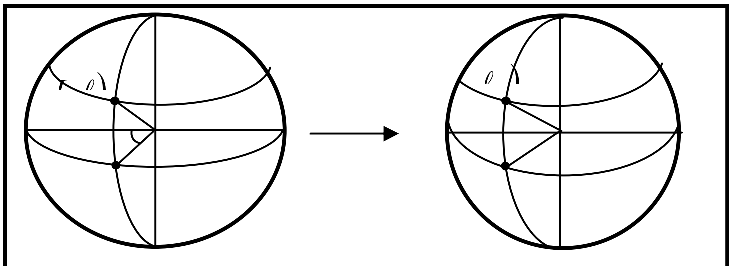

Figure (4): Conformal Projection

(L, ℓ) defines a point on the earth ellipsoid with latitude L and longitudinal ℓ

(L_c,ℓ_c) is the corresponding point on the conformal sphere with latitude L_c and longitude ℓ_c

Equation (1) shows that the earth ellipsoid and the conformal sphere having symmetrical coordinate systems which defines a correspondence between them, so that the formulae of the conformal sphere are valid for an ellipsoid with very high accuracy.

Equation (1) can be solved to produce an explicit expression for L_c and

L depending on the fact that the value of L and L_c are close to one another so that the difference ($L–L_c$) is small enough to make it possible to approximate it by developments that are limited as a function of e^2:

$$L - L_c = \left(\frac{e^2}{2} + \frac{5e^4}{24} + \frac{3e^6}{32} + \frac{281e^8}{5760} \right) \sin 2L$$
$$- \left(\frac{5e^4}{48} + \frac{7e^6}{80} + \frac{596e^8}{1520} \right) \sin 4L + ... \qquad \cdots 2a$$

$$L - L_c = \left(\frac{e^2}{2} + \frac{5e^4}{24} + \frac{e^6}{12} + \frac{13e^8}{360} \right) \sin 2Lc +$$
$$\left(\frac{7e^4}{48} + \frac{29e^6}{240} + \frac{811e^6}{11520} \right) \sin 4Lc + ... \qquad \cdots 2b$$

Note: Equation 2a is used by the application program to calculate the value of Lc when L is known

Equation 2b is used by the application program to calculate the value of L when L_c is known.

3-2 Projection of a Sphere onto a Plane:

Plane which is tangent to the sphere at the center of the system and its antipode is used for this projection as shown below in figure (5).

The following projection expressions ensure that for every point (L_c, ℓ_c) on the conformal sphere there is a corresponding point (U, V) on the tangent plane [5], [6].

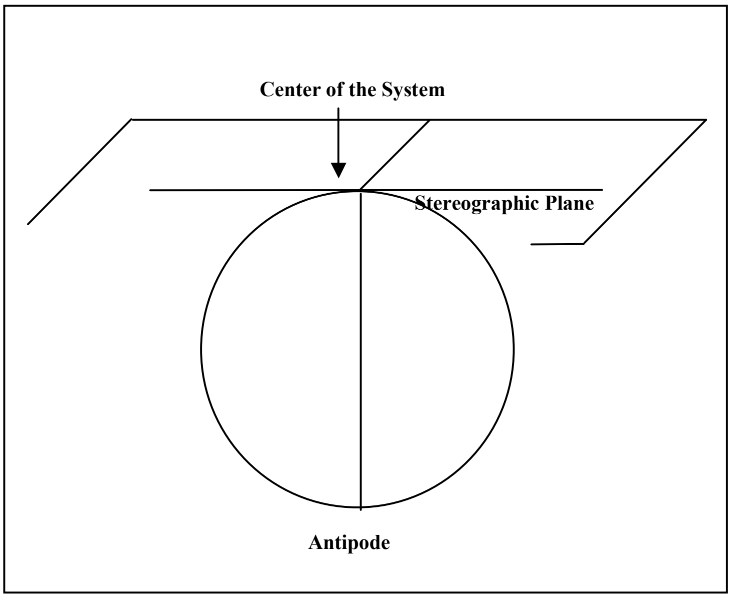

Figure (5): Stereographic Plane (Tangent

$$U = 2R \frac{\sin(\ell - \ell_0)\cos Lc}{1 + \sin Lc \sin Lco + \cos L_C \cos Lco.\cos(\ell - \ell_0)} \qquad \ldots 3a$$

$$V = 2R \frac{\sin L_c \cos L_{c0} - \cos L_c \sin L_{C0} \cos(\ell - \ell_0)}{1 + \sin Lc \sin Lco + \cos L_C \cos Lco.\cos(\ell - \ell_0)} \qquad \ldots 3b$$

where:

$R = \frac{a}{\sqrt{1 - e^2 \sin^2 Lo}} . \frac{\cos Lo}{\cos Lco}$ This is the radius of the conformal sphere which ensures a conformal projection:

$$L_{co} = L_o - \left(\frac{e^2}{2} + \frac{5e^4}{24} + \frac{e^6}{12} + \frac{13e^8}{360} \right) \sin 2_{Lo}$$
$$+ \left(\frac{5e^4}{48} + \frac{7e^6}{80} + \frac{597e^8}{1520} \right) \sin 4Lo \ + ... \qquad \ldots 4$$

a = earth radius at the equator (major axis) = 6378.137km

e = eccentricity of the earth ellipsoid = 0.0818191908

e^2 = second eccentricity of the earth ellipsoid = 0.00669437999

The tangent point (L_{co}, ℓ_o) between the sphere and the stereographic plane will be a fixed point from which the geographical map will be centered, this point form the origin (0.0) of the stereographic coordination. This also means that the point with latitude (L_c) and longitude (ℓ_c) will be corresponding to the origin (0,0) of the stereographic plane. (see Fig. 6).

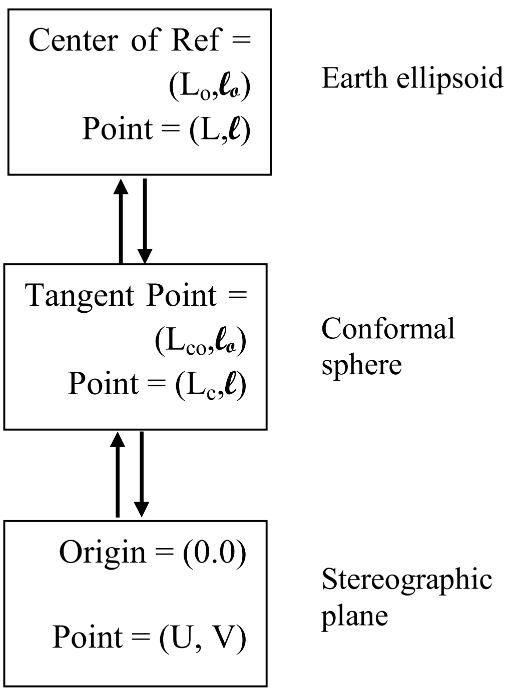

Figure (6): Schematic Diagram of the transformation process

The position (L_o, ℓ_o) is chosen to be the center of reference, which will from the origin of the stereographic coordinates.

3-3 Inverse Formulae:

If a point (U,V) on a stereographic plane is selected, a calculation of conformal latitude and longitude corresponding to this point can be made using the following formula [7]:

$$\tan\left(\ell - \ell_o\right) = \frac{U}{R\cos Lco\left(1 - \frac{d^2}{4R^2}\right) - V\sin Lco} \quad \ldots 5a$$

$$\sin\left(Lc - Lco\right) = \frac{1}{R\left(1 + \frac{d^2}{4R^2}\right)} \times \left(V - U\sin Lco . \tan\left(\frac{\ell - \ell o}{2}\right)\right) \quad \ldots 5b$$

where $(Lco, \ell o)$ is the conformal latitude and longitude of the center of reference, when the conformal latitude (L_c) is computed , the geodetic latitude (L) can be deduced using formula (2b):

$$L = Lc + \left(\frac{e^2}{2} + \frac{5e^4}{24} + \frac{e^6}{12} + \frac{13e^8}{360}\right)\sin 2Lc + \left(\frac{7e^4}{48} + \frac{29e^6}{240} + \frac{811e^8}{11520}\right)\sin 4Lc + \ldots \quad \ldots 6$$

4. Results and Discussion

The MLM is designed carefully to fulfill some requirements, though it is still at a preliminary stage, it can be developed to include many application related to the geodesy science and GIS application. The

MLM implements the inverse calculation of a map projection as well, this is necessary to provide the spatial information the other way round (i.e. from the plane to the earth ellipsoid). The results achieved by this work are shown in figures (7 through 12) which demonstrates the concept of a multilayer mapping with spatial reference.

Figure(7) display of the grid of geographical angles (latitude and longitude) of the study map which includes the border layer with the readout of the spatial coordinates shown on the top right corner of the figure.

Figure(8) displays part of the grid (zoomed) of geographical angles, the center of the map is chosen to be the country of Iraq.

Figure(9) display of the layer borders of Iraq and neighboring countries which were added to the layers shown in figure(8).

Figure(10) display of the layer of rivers which were added to the layers shown in figure(9).

Figure(11) display of the layer that contains geographical locations of the cities which were added to the layers shown in figure(10).

Figure(12) display of the layer that contains countries names which were added to the layers shown in figure(11)

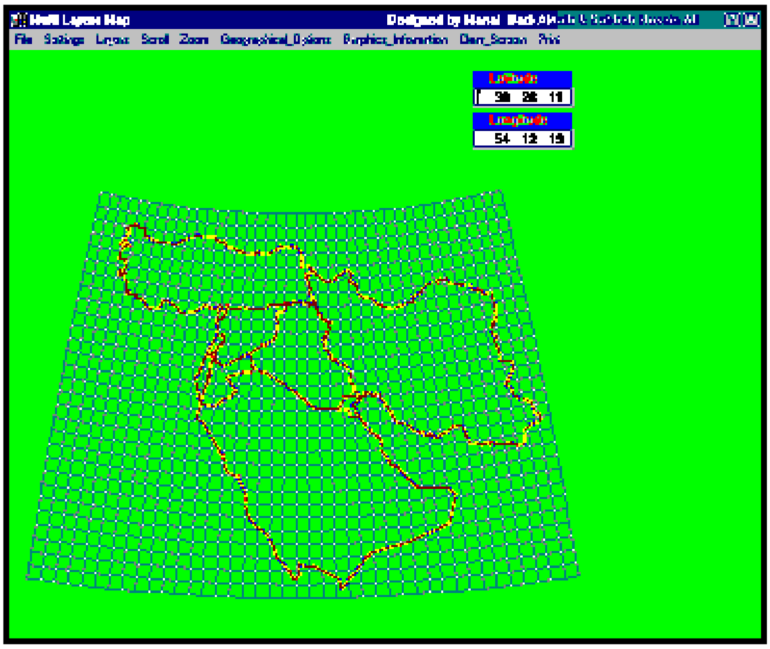

Figure (7): The grid of the study area

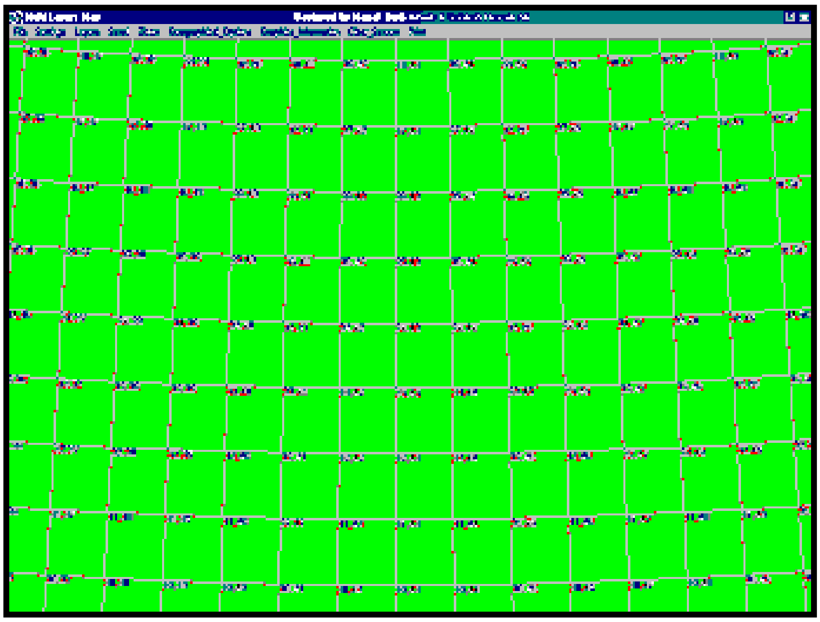

Figure (8):The grid of the angles(latitude and longitude)

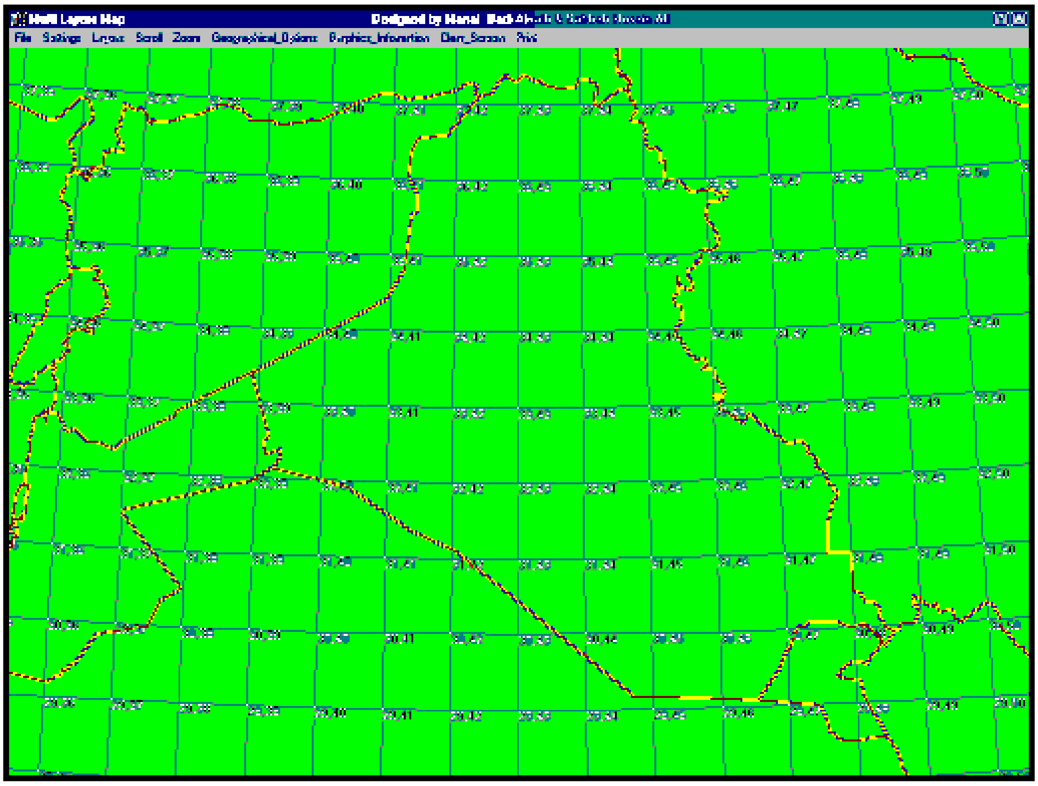

Figure (9):The layer of border

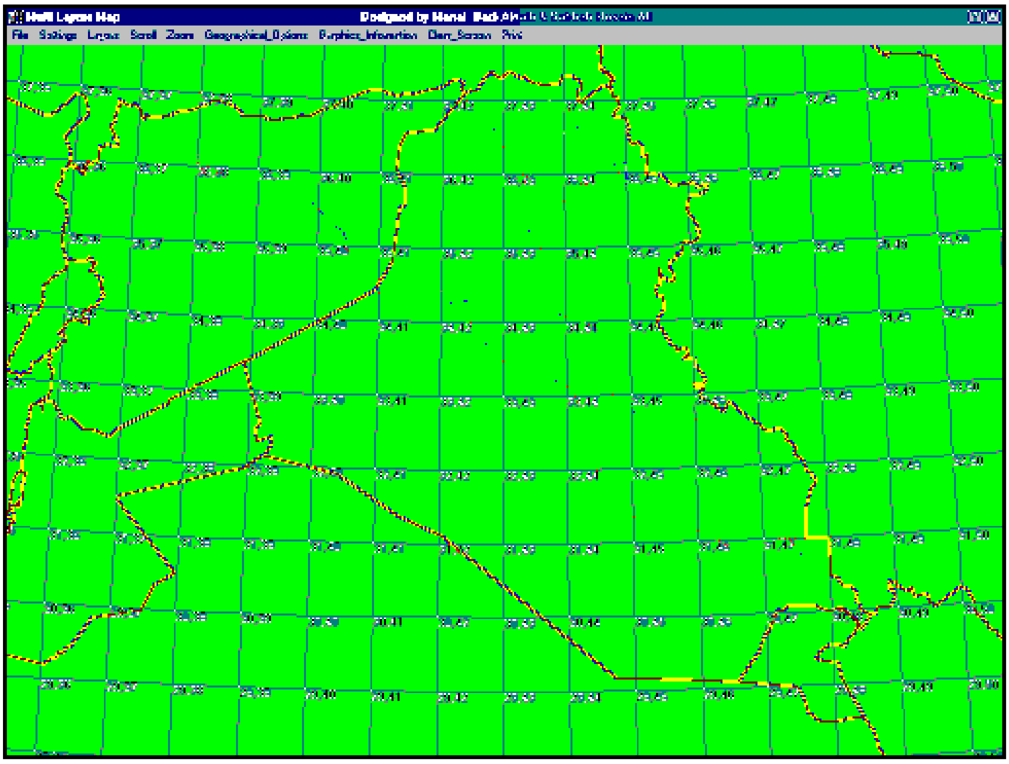

Figure (10):The layer of rivers

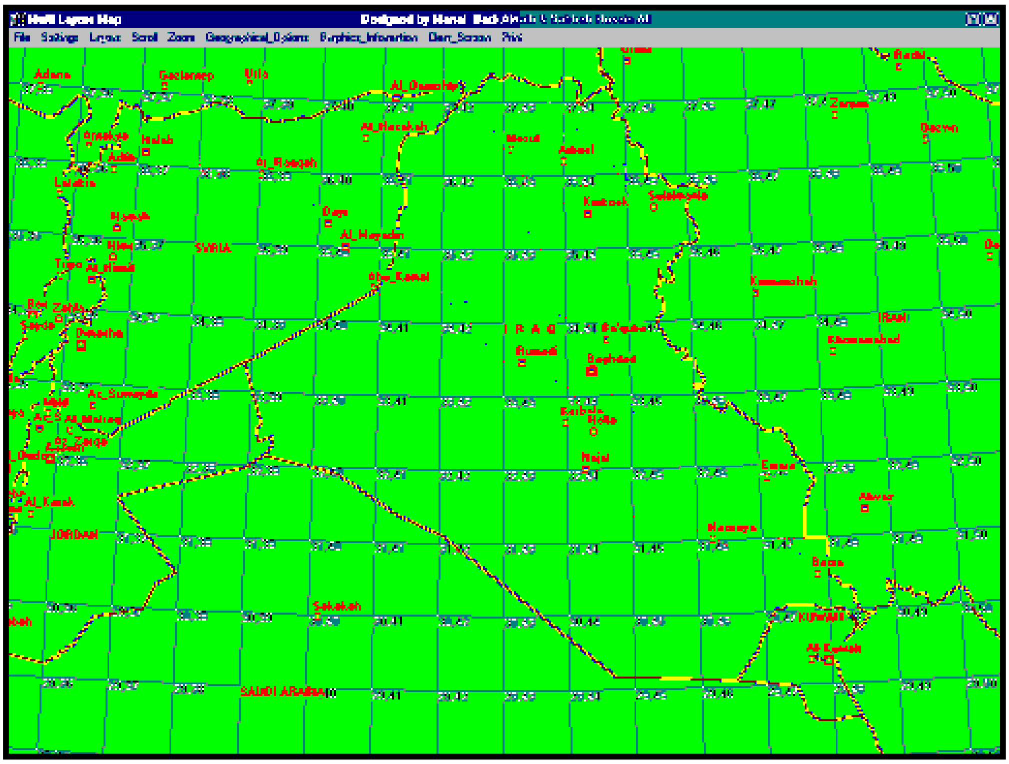

Figure (11):The layer of cities name

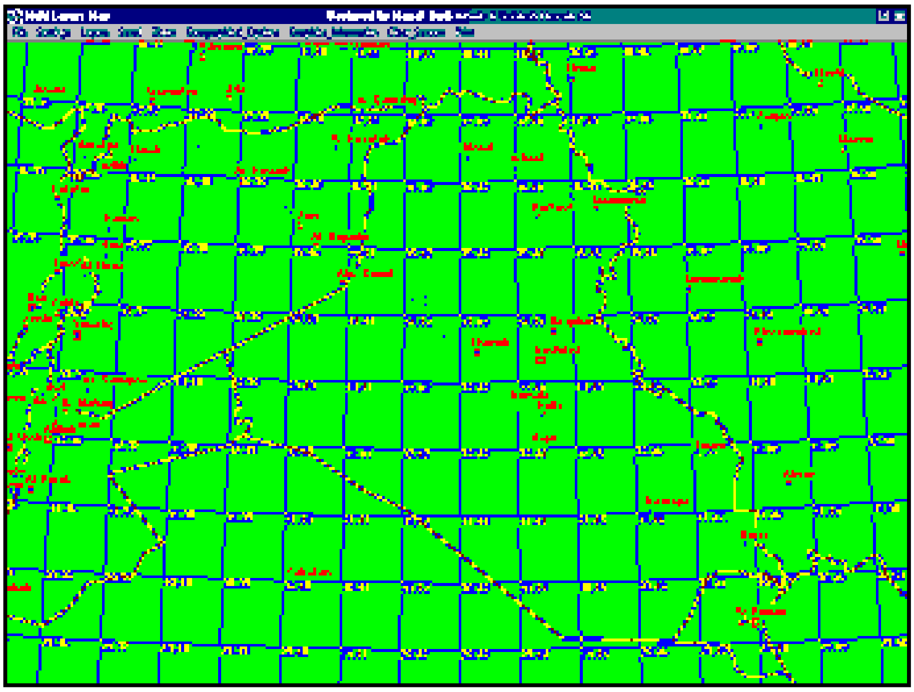

Figure (12):The layer of countries name

5. Conclusions

From the output results of the research, we conclude that designing a software application using map projection techniques which fulfills special requirements is quite possible. This possibility depends on the availability of the database, the database can be expanded to include many features according to the various requirements.

One of the advantages of the MLM that it can produce a hard copy of the multilayer map of the study area, it can be developed to produce the various information (such as contour lines) from a georeferencing database. development on software can be made to include many applications, for instance, linking a GPS as a main source input of spatial information to produce some sort of navigation, or producing a detailed information for a map of streets (local places)….etc.

Generally using any map projection techniques yields some errors in the computation of the geographical angles. The map projection technique applied in this paper has shown a small increase in the accumulated error as the distance from the origin of the system to the projected point increases, however this error is still small enough that can be neglected.

6. References

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