ADAPTIVE HYBRID-WAVELET METHOD FOR GPS/INS SYSTEM INTEGRATION

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ABSTRACT

In this paper, a technique for error estimation in a global positioning system and inertial navigation system (GPS/INS system) based on a low-cost inertial measurement unit (IMU) is offered. This technique is composed of Wavelet Transform (WT) and Adaptive Fuzzy System (AFS). The wavelet decomposition is used to de-noise the position and velocity components of the GPS and INS outputs. An AFS is introduced in this paper to estimate the position and velocity errors in the integrated system in order to provide accurate navigation information about the moving vehicle.

Several data sets are processed in this paper, where the simulation results are based on MatLab7 programming language. Six AFS networks are used to process the position and velocity components. The average error value per sample was 0.0142, 0.0443, and 0.0108 m for position in X, Y, and Z axes respectively and 0.0077, 0.0223, and 0.0269 m/s for velocity in North, East, and Down directions respectively.

Keywords: *GPS, Inertial navigation system, Integrated Systems, Wavelet Transform, Adaptive fuzzy system*

1. INTRODUCTION

Choosing an appropriate estimation method is a key problem in developing an aided INS. Many approaches have been identified in the research on estimation methods for integrated navigation systems [6]. Most of them implement the Kalman filtering technique with its drawbacks related to the need for predefined INS error model, immunity to noise effects and observability [3].

A fundamental difference between AI-based estimation methods and the other two types is that AI-based methods do not use any mathematical models in the system dynamics and measurements. AI-based methods developed to date neglect the mathematical models of vehicle motion and measurements. Thus, AI-based methods can show superior performance when long measurement outages occur. In addition, AI-based methods are simpler in terms of design [6].

The aim of this work is to use different trajectories for a vehicle in each of the position components, X, Y, and Z-axes, and each of the velocity components, North, East, and Down directions, to evaluate the performance of the AFS in estimating the positions and velocities' errors for different trajectories.

This is done after applying wavelet multi-resolution analysis (WMRA), used in [5], to eliminate the effect of the errors that combined with the GPS/INS signals. Wavelet analysis is used to analyze and smooth the error of each component of the INS and GPS by eliminating the high frequency component from the errors then the AFS is used to predict the error of the INS to provide accurate position and velocity.

In this paper, is different in handling the deficiency in navigation systems utilizing the wavelet and adaptive fuzzy system, unlike the previous work in [5] which uses wavelet and neural techniques.

2. GPS/INS INTEGRATION USING AFS-WAVELET TECHNIQUE

The first step in implementing this technique is to construct the GPS/INS error signals. As mentioned previously, the WMRA algorithm is used to process the GPS and INS data of 15 trajectories for each position and velocity component and to output a GPS/INS error signal associated with each trajectory, i.e. for each couple of GPS data and INS data, the WMRA algorithm constructs a GPS/INS error signal. These error signals will be compared with the output of the AFS networks, i.e. they are used as target outputs to the AFS networks.

2.1 ADAPTIVE FUZZY SYSTEM STRUCTURE

The equation which represents a fuzzy logic system with center average defuzzifier, product interface rule, non-singleton fuzzifier, and bell-shaped membership function is:

$$f(\underline{x}) = \frac{\sum_{j=1}^{M} y_j \left[\prod_{i=1}^{n} \exp\left[-\left(\frac{x_i - m_{ij}}{\sigma_{ij}} \right)^2 \right] \right]}{\sum_{j=1}^{M} \left[\prod_{i=1}^{n} \exp\left[-\left(\frac{x_i - m_{ij}}{\sigma_{ij}} \right)^2 \right] \right]} \qquad (1)$$

where

- $f(\underline{x})$: Fuzzy logic system output, which represent a function to n input variables x
- x_i : Input variable in the input universe of discourse
- y_j : Center of fuzzy set F_j, which is, a point in the universe of discourse V when membership function (μ_{Fj} *(y)*)achieves its maximum value, and μ_{Fj} (y) is given by a product interface engine
- M : The number of fuzzy rules

N	:	The number of input variables
m_i, σ_i	:	The center and width of the bell-shaped function of the i^{th} input variable, respectively.

This equation can be implemented on a Forward Neural Network (FNN). This connectionist model combines the approximate reasoning of fuzzy logic into a five layer neural network structure [4].

Based on the error back propagation algorithm for multi-input single-output (MISO) system, the goal is to determine a fuzzy logic system $f(\underline{x})$ in the form of equation (1), which minimizes the error function:

$$E(k) = \frac{1}{2}(f(x(k)) - d(k))^2 \tag{2}$$

where

d(k) is the desired output at time k.

According to equation (1), if the number of rules is M, then the problem becomes training the parameters y_j, m_{ij}, and σ_{ij} such that E(k) is minimized. And based on the back propagation training algorithm the iterative equations for training the parameters y_j, m_{ij}, and σ_{ij} are [4]:

$$y_j(k+1) = y_j(k) - \eta\big(f(\underline{x}(k)) - d(k)\big)\frac{1}{D} z_j \tag{3}$$

$$m_{ij}(k+1) = m_{ij}(k) - 2\eta \frac{z_j}{D}\big(f(\underline{x}(k)) - d(k)\big)\cdot\big(y_j(k) - f(\underline{x}(k)\big) \cdot \left(\frac{x_i^2(k) - m_{ij}}{(\sigma_{ij})^2}\right) \tag{4}$$

$$\sigma_{ij}(k+1) = \sigma_{ij}(k) - 2\eta \frac{z_j}{D}\big(f(\underline{x}(k)) - d(k)\big)\cdot\big(y_j(k) - f(\underline{x}(k)\big) \cdot \left(\frac{(x_i^2(k) - m_{ij})^2}{(\sigma_{ij})^3}\right) \tag{5}$$

where

$$z_j = \prod_{i=1}^{n} \exp\left[-\left(\frac{x_i^2 - m_{ij}}{\sigma_{ij}}\right)^2\right] \tag{6}$$

D: The denominator of equation (1).

η : The learning rate.

Equations (3), (4), and (5) perform an error back propagation procedure.

2.2 CONSTRUCT INS/GPS ERROR SIGNAL STAGE [5]

In this stage, an INS/GPS error signal is constructed and the following three topics are very important in constructing the INS/GPS error signal using the WMRA algorithm.

Selection of the Appropriate Wavelet Level of Decomposition (LOD)

As discussed in [5], after applying many levels of decomposition it was found that the appropriate LOD varies for each component of position and velocity. This depends on the INS/GPS error, which is nearly equal to the real INS-error. Table I shows two cases of INS data (best and worst case) selected at the end of the vehicle's journey.

Also, it is unnecessary to increase the order of LOD because the features of the INS/GPS-error will disappear. In other words, it can't be used to model the INS error because the resulting error (INS/GPS error) will not equal the desired INS-error. On the other hand it must be mentioned that the main GPS errors can be denoised by wavelet denoising unlike the INS error where some of the error can be eliminated by wavelet denoising (optimal low pass filtering). Such error is called short-term error and the other part of the INS error is called long-term error. The latter is reduced by GPS/INS integration, which is accomplished by the multi-resolution algorithm. The output of the multi-resolution for the GPS and INS is subtracted to obtain the INS/GPS error that can not be eliminated by the wavelet denoising technique. This INS/GPS error can be used for AFS modeling in order to cancel its effect as will be described latter.

Table I: Multi-Resolution Algorithm Application to Obtain INS/GPS Standard Deviation Error

Types of data	Components	Direc-tion	INS-Error	INS/GPS error	LOD
Best INS data	Position (m)	X-axis	1.3394	1.3960	10
		Y-axis	1.2884	1.3561	11
		Z-axis	0.0418	0.0659	15
	Velocity (m/s)	North	0.0023	0.0032	19
		East	0.0618	0.0863	17
		Down	0.0016	0.0020	22
Worst INS data	Position (m)	X-axis	92.2495	91.5469	2
		Y-axis	238.8248	229.4452	1
		Z-axis	113.4915	115.2453	1
	Velocity (m/s)	North	2.8079	4.0708	10
		East	7.8505	10.7992	11
		Down	4.1089	5.8583	11

Selection of the Appropriate Filter

The wavelet transform has a flexible feature of using a variety of filters that differ by their coefficients. The corresponding type of filter to the lowest standard deviation of INS/GPS error value is the perfect filter to be used. It should be mentioned that all calculations to chose the best filter to be used for each component of position and velocity is performed for first LOD.

Table II shows the best filters to the position and velocity components after use all types of wavelet filters for best and worst INS data.

Table II: Results of Using Different Types of Wavelet Filters for Best and Worst INS Data

Filter	Standard Deviation for 1st LOD of INS/GPS Error					
	Position (m)			Velocity (m/s)		
	X-axis	Y-axis	Z-axis	North	East	Down
Best	Db4	Db9	Db6	Bior5.5	Bior2.2	Coif2
Worst	Db10	Bior2.2	Db4	Bior5.5	Bior2.2	Coif2

Thresholding Algorithm in Wavelet Coefficients

The thresholding procedure allows for cutting off some of the noise in the error signal and improving its signal-to-noise ratio so that it can be efficiently modelled using AFS. In this paper, soft thresholding is applied only to the details coefficients. The thresholding technique is standard and can be reviewed in [1] and [2].

2.3 Adaptive Fuzzy System Training Stage

The next step is the training of the AFS networks (which is done while the satellite signal is available). Six networks are used to handle each one of the position and velocity components separately. The inputs to each network are the INS data (position or velocity component) and the instantaneous time (the time is counted once the system is turned on); the output of each network is the estimated INS error for the input component, as shown in Fig. 1.

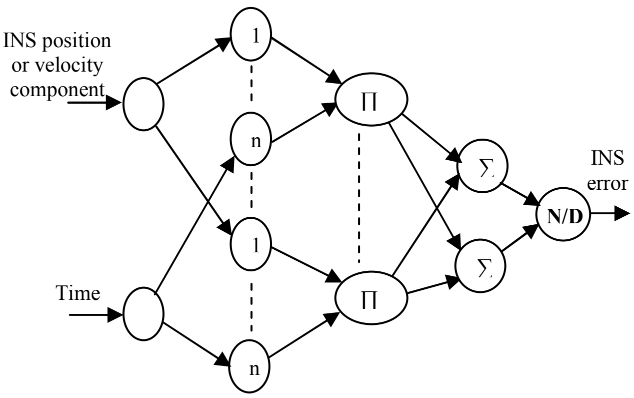

Figure (1): The Architecture of an AFS network for each component of position and velocity.

The error resulted from comparing the network output and the GPS/INS error signal is fed to the network which adjusts its parameters in a way to minimize the mean square value of the error. The parameters of the AFS network that are computed during the training stage are y, m, and σ. These parameters are updated according to equations (3), (4), and (5). The computations of these parameters are repeated until the optimal values are achieved which correspond to the minimum mean square error. The optimal values of m, y, and σ reached at the end of the training stage are saved to be used later in the testing stage, as shown in Fig. 2.

As mentioned, each component of position and velocity has its own network. To start the training, the networks need to be initialized with the number of epochs, the value of the learning rate, the number of fuzzy rules (M), and the parameters (m, y, and σ). These initial values are selected by trial-and-error. Appropriate selection of the initial values ensures good performance of the networks and converging to a minimum error value.

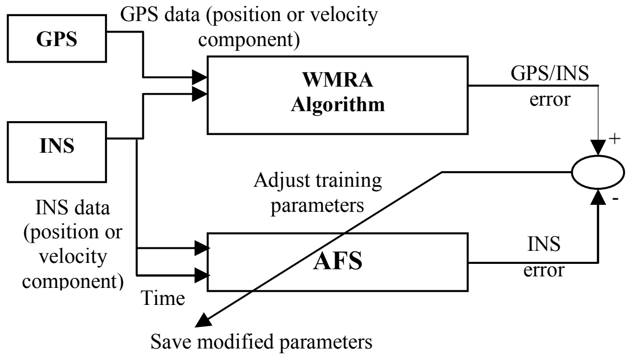

Figure (2): Block diagram of AFS-wavelet technique during training stage.

The final step in the AFS-wavelet technique is the testing stage. After the training is completed, the network is ready to work in the testing mode. The parameters of the networks are modified during the availability of the satellite signal, i.e. in the training stage. In the case of satellite signal being blocked, the networks will use the latest modified parameters saved from the training stage to perform the prediction process.

Fig. 3 shows the operation of the networks in the testing mode. It provides a prediction of the INS error based on the INS data and the particular time instant provided at the input.

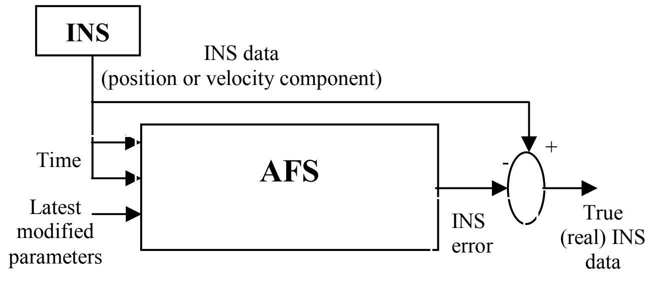

Figure (3): Block diagram of AFS-wavelet technique during testing stage.

2.4 Testing Stage

In this paper, the training of the networks was started by an attempt to use 13 GPS/INS error signals, in each of the position and velocity networks, from the 15 mentioned previously (the other two were used in the testing stage). These 13 trajectories had different shapes and were used together in the training process, i.e. one trajectory after the other. Each network was implemented many times and each time the initial values were changed in an attempt to get better performance. It was found that the value of the mean square error (MSE) was very big and often did not decrease during the training process or did not converge to the specified MSE value. Also, the saved parameters from the training stage of this attempt were used in the testing stage to evaluate their performance but undesirable results were obtained.

Hence the training process was started again with only one trajectory being used in each network at a time. This trajectory was used in the training process and the saved parameters were used in the testing mode to evaluate the performance of the AFS network in estimating the INS error of that trajectory. This process was repeated for all position and velocity components and for all trajectories. It was concluded that some of the trajectories could be used in the training process, i.e. the network can make INS error estimation for these trajectories, while the others could not be used. The number of trajectories that could be used was 10, 4, and 4 for position in X, Y, and Z axes respectively and 3, 3, and 12 for velocity in north, east, and down directions respectively. It was noticed that, for each component of position and velocity, the trajectories that could be used for the training are somehow similar (except for the velocity in east direction).

Now, the training process was started one more time. The trajectories that could be used in the training process for each position component and each velocity component, which were concluded from the previous step, are now grouped and used together in the training process, i.e. one trajectory after the other, to get the advantage of using the updated parameters (m, y, and σ) of one trajectory in the training process of the next one and so on.

Fig. 4 shows the MSE for all networks after 1000 epochs. The initial values used to obtain these results are listed in Table III. As stated early, these values are obtained by trial-and-error. The table also gives the number of trajectories used in the training process for the six networks.

Fig. 5 shows the error between the GPS/INS error (desired output) and the estimated INS error (actual output) for all networks. The maximum error was 9.9447, 16.6224, and 4.5926 m for position in X, Y, and Z axes respectively and 1.6307, 15.9221, and 0.1790 m/s for velocity in north, east, and down directions respectively.

Fig. 6 shows the error between the true INS data obtained from AFS networks and from INS algorithm.

Table IV lists the MSE obtained after 1000 epochs, the standard deviation (STD) of error between desired and actual outputs, and the STD of the actual output (estimated INS error) for all networks compared to the STD of the true INS error.

Table III: Initial Values for the Six Networks of Position and Velocity

		Position			Velocity		
		X-axis	Y-axis	Z-axis	North	East	Down
Initial Values	m	[-2,2]	[-1,1]	[-1,1]	[0, 1]	[-2,2]	[-1,1]
	y	[-1,1]	[-5,5]	[-2,2]	[0, 1]	[-1,1]	[-5,5]
	σ	[-1,1]	[.001, 1]	[.01, .3]	[0, 1]	[-1, 1]	[.001, 1]
	M	10	10	10	10	10	10
	η	0.6	0.6	0.6	0.6	0.6	0.6
No. of trajectories used in training		10	4	4	3	3	12

3. CONCLUSION

The following points summarize the main conclusions of this paper:

1. The wavelet analysis was beneficial in filtering out the noise and disturbances that may exist at the INS and GPS outputs. In addition, it provides the advantage of comparing the INS and GPS position and velocity components at different levels of resolution.
2. The advantage of using a group of trajectories in the training process is that the AFS network can continue in giving estimation of the INS error if small changes happen in the specified trajectory of a vehicle.
3. The process of selecting the initial values of the parameters (*m*, *y*, and *σ*), number of rules, and value of the learning rate is done through a trial-and-error procedure and determining the appropriate settings for one trajectory may need several attempts; therefore, handling several trajectories separately can be a very long process whereas when these trajectories are handled together, one after the other, the process of selecting the appropriate initial values is done only one time.
4. The selection of *M* (number of fuzzy rules) is essential in achieving good results. It was noticed that using large number of rules results in slow training and large error values whereas the small M values lead to small error values and fast training performance.

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Table IV: Performance of Training and Testing the AFS Networks to Predict the INS Error

Types of data	Position		
	X-axis (m)	Y-axis (m)	Z-axis (m)
MSE	0.0025	0.0059	0.0033
STD of actual o/p	67.6269	52.7566	30.0830
STD of error between desired and actual o/p	3.8108	7.8913	2.0931
STD of true INS error	70.1098	59.7733	27.8414
	Velocity		
	North (m/s)	**North (m/s)**	**North (m/s)**
MSE	0.0082	0.0082	0.0082
STD of actual o/p	3.3114	3.3114	3.3114
STD of error between desired and actual o/p	0.4547	0.4547	0.4547
STD of true INS error	2.0567	2.0567	2.0567

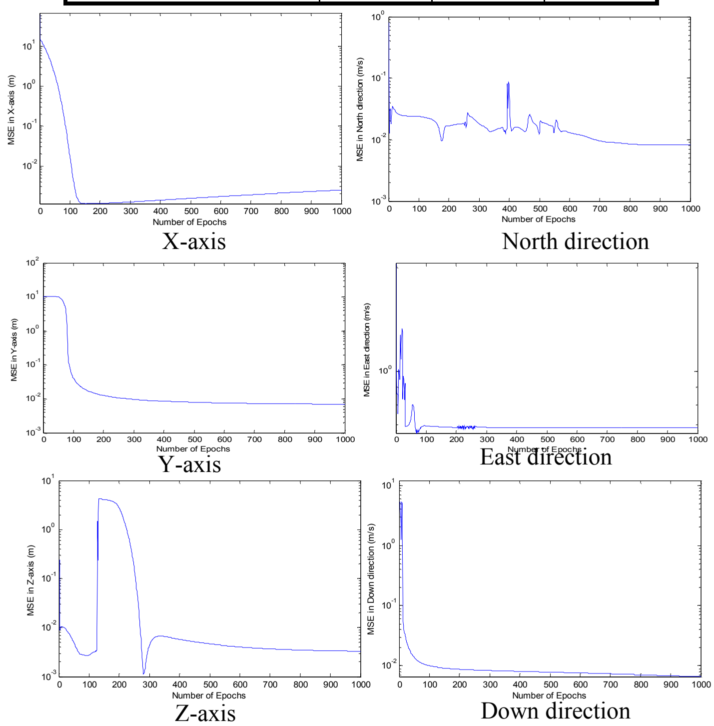

Figure (4): MSE for the six networks of position and velocity.

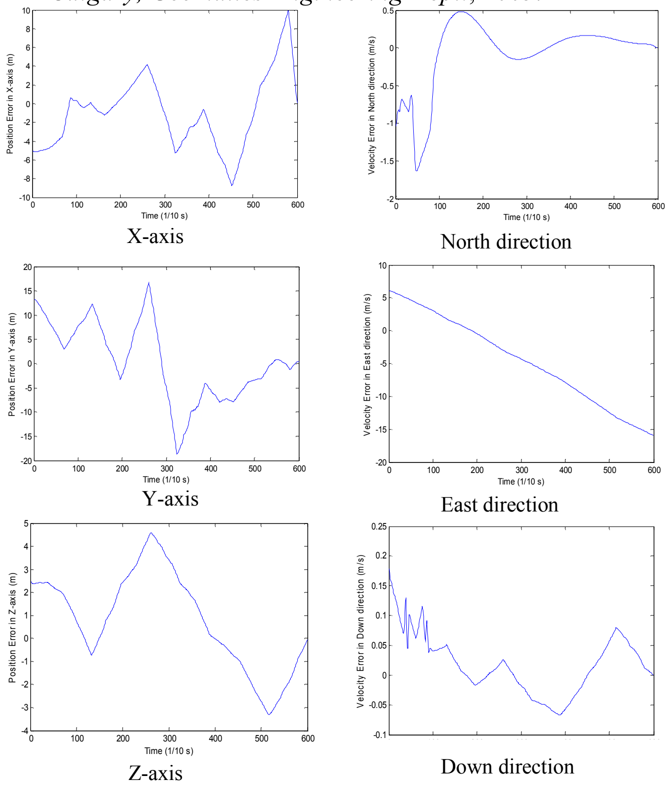

Figure (5): Error between desired and actual outputs of the AFS networks for position and velocity components.

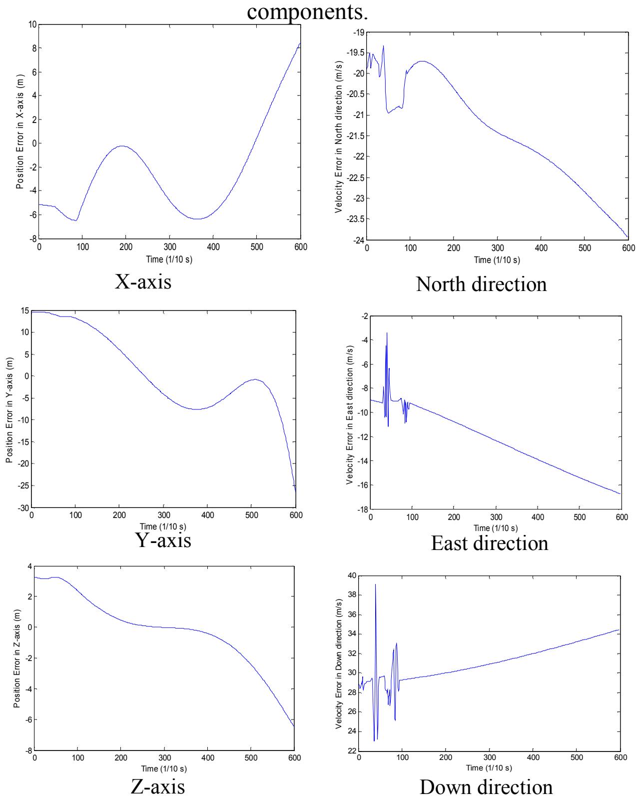

Figure (6): Error between True (Real) INS data from INS algorithm and AFS networks for all components of position and velocity.